IN THE UNITED STATES DISTRICT COURT FOR THE DISTRICT OF DELAWARE

POLAROID CORPORATION,

Plaintiff and Counterclaim Defendant,

ν.

Dated: January 11, 2008

C.A. No. 06-738-SLR

HEWLETT-PACKARD COMPANY,

Defendants and Counterclaim Plaintiff.

REDACTED

DECLARATION OF WILLIAM J. MARSDEN, JR. IN SUPPORT OF DEFENDANT HEWLETT-PACKARD'S OPENING CLAIM CONSTRUCTION BRIEF

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- I, William J. Marsden, Jr., declare as follows:
- 1. I am an attorney with Fish & Richardson P.C., counsel for Plaintiff Power Integrations, Inc. I am a member of the Bar of the State of Delaware and of this Court. I have personal knowledge of the matters stated in this declaration and would testify truthfully to them if called upon to do so.
- 2. Attached hereto as Exhibit A is a true and correct copy of excerpts from Azriel Rosenfeld and Avinash C. Kak, <u>Digital Picture Processing</u>, (1st ed. 1976).
- 3. Attached hereto as Exhibit B is a true and correct copy of Computer Graphics Systems Development, <u>Gamma Correction Explained</u>, http://www.siggraph.org/education/materials/HyperGraph/color/gamma_correction/gamma_intro.html (downloaded January 10, 2008).
- 4. Attached hereto as Exhibit C is a true and correct copy of excerpts from the transcript to the Deposition of Donald S. Levinstone (September 26, 2007).
- Attached hereto as Exhibit D is a true and correct copy of U.S. Patent No.
 4,394,688 (filed August 25, 1981).
- 6. Attached hereto as Exhibit E is a true and correct copy of excerpts from The Oxford English Dictionary, Vol. 3 (J.A. Simpson and E.S.C. Weiner, eds., 1989).
- 7. Attached hereto as Exhibit F is a true and correct copy of excerpts from Margaret L. Lial and John Hornsby, <u>Intermediate Algebra</u> (8th ed. 2000).

I declare under penalty of perjury under the laws of the United States of America that the foregoing is true and correct.

Executed this 11th of January, 2008 at Wilmington, Delaware.

/s/ William J. Marsden, Jr.
William J. Marsden, Jr.

CERTIFICATE OF SERVICE

I hereby certify that on January 18, 2008, I electronically filed with the Clerk of

Court the foregoing REDACTED DECLARATION OF WILLIAM J. MARSDEN, JR. IN

SUPPORT OF DEFENDANT HEWLETT-PACKARD'S OPENING CLAIM

CONSTRUCTION BRIEF using CM/ECF which will send electronic notification of such

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William J. Marsden, Jr. (#2247)

Exhibit A

DIGITAL PICTURE PROCESSING

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Computer Science and Applied Mathematics

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Azizi Resenisid and Arivasi C. Kak

Digital Picture Processing

Digital Picture Processing

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This is a volume in COMPUTER SCIENCE AND APPLIED MATHEMATICS A Series of Monographs and Textbooks

Editor: WERNER RHEINBOLDT

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A Subsidiary of Harcourt Brace Jovanovich, Publishers

considers nongeometrical picture properties (e.g., texture) and the description of pictures in terms of parts, properties, and relationships.

In the remainder of this chapter, we discuss the relationship between continuous and discrete pictures, and the representation of (discrete) pictures in a digital computer. We also give a brief guide to the picture-processing literature. In the next chapter, we review various useful mathematical tools.

1.2 PICTURES AND THEIR COMPUTER REPRESENTATION

1.2.1 Pictures as Functions

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Informally, a *picture* is a flat object whose brightness or color may vary from point to point. This variation can be represented mathematically by a function of two spatial variables. When color is involved, the function should be regarded as vector valued, or several functions should be used; see Section 3.3. In this book, however, we will be dealing almost exclusively with blackand-white pictures in which there can be shades of gray, but no color. Such a picture can be represented by a single real-valued function, say f(x, y). The value of this function at a point will be called the *gray level* or *brightness* of the picture at that point.

It is customary to assume that the functions that represent pictures are analytically well behaved—so that, for example, these functions are integrable, have invertible Fourier transforms, etc. It is usual also to regard these functions as having values that are nonnegative and bounded, i.e., $0 \le f(x, y) \le M$ for all x, y.

1.2.2 Pictures as Arrays

When a picture is *digitized* (Chapter 4), a *sampling* process is used to extract from the picture a discrete set of real numbers ("samples"), and a *quantization* process is applied to these samples to yield numbers having a discrete set of possible values. In most practical situations, the samples are the values of the picture at a discrete, usually regularly spaced, set of points—or, more realistically, averages of the values taken over small neighborhoods of such points. Such a set of samples can be represented, for computer-processing purposes, as a (rectangular) *array* of real numbers. The samples are usually quantized to a set of equally spaced gray level values (see Section 4.3). If the unit of measurement is suitably chosen, these values can be taken to be integers; thus a digitized picture, or *digital picture*, can be regarded as an

1.2 Pictures and Their Computer Representation

integer array.[†] The elements of a digital picture array are called *picture* elements, pixels, or pels, or sometimes just "points."

The most common method of picture sampling is to use a regularly spaced square array of points, i.e., points (md, nd) whose coordinates are multiples of some unit distance d. In such an array, every point has two kinds of neighbors—four horizontal and vertical neighbors (above, below, to the left, and to the right), at distance d from the point, and four diagonal neighbors at distance $d\sqrt{2}$. (Some of the complications introduced by the existence of these two types of neighbors will be discussed in Chapter 9.) Of course, at the edges of the array, some of these neighbors will not exist.

Another possibility is to use a regular hexagonal array of sample points; here each point has six neighbors, all at the same distance from it. One can obtain a similar kind of neighborhood from a square array by shifting, say, odd-numbered rows d/2 to the right (see Fig. 1). In fact, rather than do the



Fig. 1 "Hexagonal" array obtained by shifting odd-numbered rows of a square array.

actual shifting, one can simply stipulate that, for a point p on an odd-numbered row, the neighbors of p are its four horizontal and vertical neighbors together with just two of its diagonal neighbors, namely those to the northeast and southeast; and similarly for p on an even-numbered row, but this time allowing only the northwest and southwest diagonal neighbors.

The picture arrays that are processed in practice can be very large. For example, suppose that we want to sample and quantize an ordinary television image finely enough so that it can be redisplayed without noticeable degradation. Then we should use an array of about 500×500 samples, and we should quantize the samples to at least 30-50 discrete gray levels, i.e., a quarter of a million 5- or 6-bit numbers (see Chapter 4). In many cases it is necessary to handle even larger arrays when digitizing high-resolution photographs; and it is often desirable to use finer quantization, to 8 or even 10 bits. (It is customary to use a power of 2 as the number of quantization levels; and one often

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[†] In particular, if there are just two gray levels, "black" and "white," we usually represent them by 0 and 1, so that the picture becomes a Boolean array.

"exposure" when the picture was originally recorded. The second type is gray scale transformation, which has the aim of changing the gray scale in a uniform way throughout the picture, or perhaps throughout some region of the picture, usually in order to increase contrast and thereby make details of the picture more easily visible. An important special case of this is histogram modification, in which a gray scale transformation is used to give the picture a specified distribution of gray levels.

6.2.1 Gray Level Correction

A picture recording system should map object brightness into picture gray level in a monotonic fashion, and this mapping should ideally be the same at every point of the picture. In practice, however, the mapping often varies from point to point. For example, light passing along the axis of an optical system is generally attenuated less than light that passes through the system obliquely. Thus when an image is formed by the system, the parts of the image far from the axis will be attenuated relative to the parts near the axis; this phenomenon is called "vignetting." As another example, the photocathode of a vidicon may not be equally sensitive at all points; thus in a picture obtained using the vidicon, equal gray levels may not correspond to equally bright points in the scene.

If we can determine the nonuniform "exposure mapping" that produced a given picture, it is then straightforward to correct the picture, by changing the gray level of each point, to achieve the effect of a uniform mapping. Specifically, suppose that we describe the nonuniform mapping by an expression of the form

$$g(x, y) = e(x, y) f(x, y)$$

where f(x, y) is the ideal gray level that should have resulted at picture point (x, y) had the "exposure" been uniform [i.e., f(x, y) is the desired monotonic function of the object brightness at the object point corresponding to (x, y)], and g(x, y) is the actual gray level at point (x, y), due to the nonuniform mapping. To determine the function e(x, y), we can calibrate the picture recording system by taking a picture of a uniform field of known brightness. For such a field, f is a known constant, call it c. If $g_c(x, y)$ is the picture of the uniform field, we have $e(x, y) = g_c(x, y)/c$. Once we know e(x, y), we can correct any picture g(x, y) obtained by the system (as long as the calibration does not change!), since

$$f(x, y) = g(x, y)/e(x, y) = cg(x, y)/g_c(x, y).$$

In any practical gray scale, the range $[z_1, z_K]$ of gray levels used is limited

by the dynamic ranges of the available display devices. The correction operation just described may give rise to gray levels outside the allowable range. One way of handling this situation is simply to change any gray level less than z_1 to z_1 , and any level greater than z_K to z_K . Another possibility is to shrink and/or shift the scale of the corrected gray levels until it falls within the allowable range. (Methods of doing this will be described in Section 6.2.2.) It should also be noted that even if the uncorrected gray levels were quantized to a discrete set of values $z_1, ..., z_K$, the corrected levels—even though they lie in the range $[z_1, z_K]$ —need not have these discrete values. Thus gray level correction of a quantized picture must be followed by requantization.

6.2.2 Gray Scale Transformation

We next consider gray scale transformations that are the same at every point of the picture (or a region), rather than varying from point to point. Such a transformation can be expressed as a mapping from the given gray scale z into a transformed gray scale z', i.e.,

$$z' = t(z)$$

We shall assume that in both the old and new gray scales, the allowable range of gray levels is the same, i.e., $z_1 \le z \le z_K$ and $z_1 \le z' \le z_K$. We can now discuss how to design gray scale transformations that have enhancing effects, e.g., that increase contrast.

It is easy to increase the contrast of a picture if the picture does not occupy its full allowable gray level range. (This can happen if the picture recording device used had a smaller dynamic range than $[z_1, z_K]$, or if the picture was originally "underexposed.") Suppose that, in the given picture f, we have $a \le z = f(x, y) \le b$ for all x, y, where [a, b] is a subinterval of $[z_1, z_K]$. Let

$$z' = \frac{z_K - z_1}{b - a}(z - a) + z_1 = \frac{z_K - z_1}{b - a}z + \frac{z_1b - z_Ka}{b - a}$$

This simple linear gray scale transformation stretches and shifts the gray scale to occupy the full range $[z_1, z_K]$.

A similar approach can be used if *most* of the gray levels of the given picture lie in the subrange [a, b]. In this case, we can use the transformation

$$z' = \begin{cases} \frac{z_K - z_1}{b - a}(z - a) + z_1 & \text{for } a \leq z \leq b \\ z_1 & \text{for } z < a \\ z_K & \text{for } z > b \end{cases}$$

This piecewise linear transformation stretches the [a, b] interval of the

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More generally, we can stretch selected regions of the gray scale, at the cost of compressing other regions, if we want to bring out detail in the stretched regions, and we do not care about loss of information in the compressed regions. As a simple example, suppose that the gray scale range is [0, 30]; then the transformation

$$z' = \begin{cases} z/2 & \text{for } z \le 10\\ 2z - 15 & \text{for } 10 \le z \le 20\\ (z/2) + 15 & \text{for } 20 \le z \le 30 \end{cases}$$

compresses the gray scale by a factor of 2 in the ranges [0, 10] and [20, 30] (on the original), while expanding it by a factor of 2 in the range [10, 20].

Exercise 6. Give the equations for the transformation that stretches gray scale range [0, 10] into [0, 15], shifts range [10, 20] to [15, 25], and compresses range [20, 30] into [25, 30].

The gray scale transformations that we use can, of course, be smooth rather than piecewise linear. We can implement any desired mathematical transformation z'=t(z)—quadratic, logarithmic, or completely arbitrary—subject only to the restriction that the results lie in the allowable range $[z_1, z_K]$ —i.e., that $z_1 \le t(z) \le z_K$ for all $z_1 \le z \le z_K$. This can always be achieved by incorporating a suitable shift and scale factor into the transformation. We can do this as follows: given any t(z), let $t_1 = \min t(z)$ and $t_K = \max t(z)$ for $z_1 \le z \le z_K$. Then the modified transformation t' defined by

$$t'(z) = \frac{z_K - z_1}{t_K - t_1} [t(z) - t_1] + z_1$$

satisfies $z_1 \le t'(z) \le z_K$ for all $z_1 \le z \le z_K$.

Exercise 7. Give the equations for a transformation t for which t(z) is a linear function of $\log z$ over the range $10 \le z \le 100$.

Two simple examples of contrast-stretching gray scale transformations are shown in Figs. 7 and 8. The graph of such a transformation, t(z) as a function of z, is a useful aid in visualizing the effect of t. In ranges where this graph has slope <1, contrast is compressed, while slope >1 implies that contrast is stretched.

The examples of transformations considered up to now have all been monotonic [i.e., $z' \le z''$ implies $t(z') \le t(z'')$] and continuous. It is sometimes

6.2 Gray Scale Modification

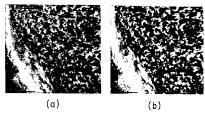


Fig. 7 Contrast stretching. (a) "Underexposed" picture (all gray levels in middle half of range). (b) Result of stretching the gray scale of (a) by a factor of 2.

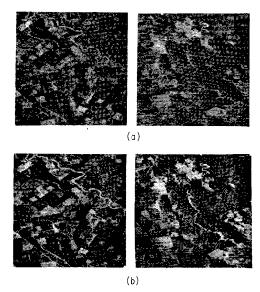


Fig. 8 Partial contrast stretching. (a) Input pictures. (b) Result of stretching the middle third of part (a)'s gray scale by a factor of 2 while compressing the upper and lower thirds by a factor of 2.

of interest, however, to consider nonmonotonic discontinuous transformations in which different ranges of input gray levels are mapped into the same range of output levels. As a simple example, if the input range is [0, 30], the discontinuous transformation

$$t(z) = \begin{cases} 3z, & 0 \le z \le 10 \\ 3(z-10), & 10 < z \le 20 \\ 3(z-20), & 20 < z \le 30 \end{cases}$$

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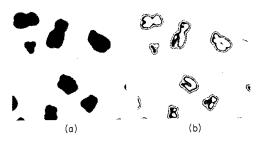


Fig. 9 Discontinuous, nonmonotonic contrast stretching. (a) Input picture (same as Fig. 2b of Chapter 8). (b) Result of stretching each third of part (a)'s nonzero gray levels to cover the full gray level range.

maps each third of this range into [0, 30]. Within these subranges, we have threefold contrast stretching; but as we cross from one range to another, strong false contours are introduced, as illustrated in Fig. 9.[†]

This discussion has treated the input gray scale as though it were continuous rather than quantized. If it is, in fact, quantized, say to the K discrete values $z_1, ..., z_K$, then any gray scale transformation t can only produce K discrete output values $t(z_1), ..., t(z_K)$, some of which may be equal.[‡] Nevertheless, contrast stretching can be advantageous, since it enables us to space (some of) the output values farther apart, so that they become more clearly distinguishable. In a well-designed gray scale, consecutive levels z_i, z_{i+1} should not be easily distinguishable; in fact, if they were, we could never get the impression of continuously varying gray levels in a picture. Thus detail that is not sharply delineated in the input is enhanced in the output when we spread the gray levels. Note, however, that once the levels have been spread far enough apart to become easily distinguishable, there is little to be gained in spreading them still farther apart, since all the available information (in the gray level range that has been stretched) has already been brought out. Note also that in ranges where the gray scale is compressed, information is lost, since when we requantize, several input levels may be mapped into the same output level.

If a picture's gray scale has been distorted by a known gray scale transformation z' = t(z), the distortion can in principle be corrected by applying

6.2 Gray Scale Modification

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the inverse transformation $z = t^{-1}(z')$. For example, this approach can be used to correct for the effects of nonlinearities in the gray scales of display devices, or of recording media such as photographic film. In practice, of course, implementation of t^{-1} may be subject to the difficulties previously discussed.

6.2.3 Histogram Modification

Given a picture f, let $p_f(z)$ denote the relative frequency with which gray level z occurs in f, for all z in the gray level range $[z_1, z_K]$ of f. The graph of $p_f(z)$ as a function of z, normalized so that $\int_{z_1}^{z_K} p_f(z) dz$ is equal to the area of f, is called the histogram of f. If f is quantized, and has gray levels z_1, \ldots, z_K , its histogram can be represented as a bar graph having K bars. As we shall see in Chapters g and g and g and g are the histogram of g can provide useful information about how to segment g into parts, and it also serves as a basis for measuring certain textural properties of g. Note that for any g and g and g because g is populated, i.e., what fraction of g is points have their gray levels in that range.

In this section we describe how to transform a picture's gray scale so as to give the picture a specified histogram q(z). The following are some cases where this type of transformation might be required:

- (a) If we want to quantize a picture f to K discrete levels in such a way as to minimize quantization error (Section 4.3.1), the K levels should be spaced close together in heavily populated regions of f's gray scale, but they can be farther apart in sparse regions. One way of doing this is to pick the levels at the midpoints between the successive K-tiles of f's histogram; this means that when we quantize, just one Kth of the points of f will be quantized to each of the levels. This "tapered" quantization scheme thus transforms f's histogram into a "flat" bar graph in which all the bars have equal height.
- (b) Suppose that we want to compare two pictures f_1 and f_2 in order to (say) detect differences between them. If the pictures were obtained under different lighting conditions, we must somehow compensate for this, since otherwise they will have different gray levels at every point even if they are pictures of the same scene. One way to carry out this compensation might be to transform the gray scale of f_1 so that its histogram matches that of f_2 , or to transform both pictures so that they have some standard histogram. (On picture matching see Section 8.3.)
- (c) We often want to measure certain properties of a picture f in order to classify or describe it. If these properties depend on the gray levels that are present in f, their values will be sensitive to the lighting conditions under which f was obtained. This sensitivity can be reduced by "normalizing" f so that it

[†] Another important "contrast-stretching" technique is to map the gray levels of a picture into *colors*—e.g., low levels into shades of red; higher ones into orange shades; still higher ones into yellow, and so on. This can greatly enhance the visibility of detail in the picture, because the eye is more sensitive to differences in hue than it is to differences in brightness.

^{*} See, however, Section 6.2.3, where we discuss how to map a single input gray level into more than one output level.

Exhibit B

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CGSD - Gamma Correction Explained

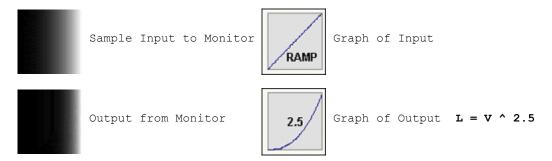
What is Gamma Correction?

In the field of Computer Graphics, one often hears the phrase "gamma correction." What is this strange sounding thing and why does it matter to you?

Gamma correction matters if you have any interest in displaying an image accurately on a computer screen. Gamma correction controls the overall brightness of an image. Images which are not properly corrected can look either bleached out, or too dark. Trying to reproduce colors accurately also requires some knowledge of gamma. Varying the amount of gamma correction changes not only the brightness, but also the ratios of red to green to blue. (Example of this color phenomenon). Gamma correction also plays a big role in making images for the WWW.

To explain gamma correction we will begin with where you are looking - your computer monitor.

Almost every computer monitor, from whatever manufacturer, has one thing in common. They all have a intensity to voltage response curve which is roughly a 2.5 power function. Don't be afraid, this just means that if you send your computer monitor a message that a certain pixel should have intensity equal to x, it will actually display a pixel which has intensity equal to $x ^2$.5 Because the range of voltages sent to the monitor is between 0 and 1, this means that the intensity value displayed will be less than what you wanted it to be. $(0.5 ^2.5 = 0.177)$ for example) Monitors, then, are said to have a gamma of 2.5

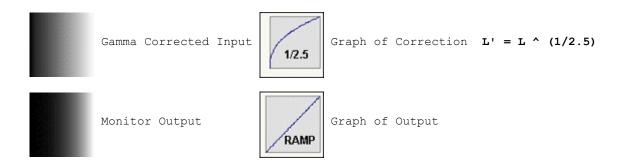


Note: The grayscale images will not look very good on 8-bit color computers. Also the gamma correction for these images is 1.0 so they were actually designed to be viewed on a system such as a Sun or a PC with no hardware correction. They may appear brighter on other systems. The important thing here is the relative difference that you see.

To correct this annoying little problem, the input signal to the monitor (the voltage) must be "gamma corrected".

The solution, fortunately, is a simple one. Since we know the relationship between the voltage sent to the monitor and the intensity which it produces, we can correct the signal before it gets to the monitor. The signal is adjusted so that it is essentially the complement of the curve shown above. There are other considerations as well when one speaks of a "correct" image. These include the ambient light in a room where the computer is, the brightness and contrast settings on the monitor, and finally personal taste.





If gamma correction is done properly for the computer system, then the output should accurately reflect the image input.

Note that the task of gamma correction is accomplished by raising the input value to the 1/2.5 power. This is referred to as a gamma correction of 2.5. because we are correcting the input for a monitor whose gamma is 2.5

Real World Application

In the real world, it isn't quite this simple, especially when an image needs to look good on different systems, or platforms.

As we mentioned above, most monitors work in about the same way with respect to gamma correction. Most computers, or more specifically, most computer systems, do not work in exactly the same way, however.

By computer systems we mean everything from the software that is running (like Netscape) to the graphics cards installed, to the standard hardware on the motherboard. Different computers do different things and many "systems" have different configurations of all of the above things.

Macintoshes, for example, have partial gamma correction built-in to their hardware. Silicon Graphics computers also have built-in gamma correction, but it is different from the Macintosh. Suns and PCs have no standard built-in gamma correction but some graphics cards installed in these computers may provide this functionality.

System Gamma

The idea of system gamma, is the gamma correction that should be applied in the software to reproduce an accurate image on the monitor for an uncorrected image on a particular computer "system."

Macintosh

The Macintosh has built-in gamma correction of 1.4. This means that after the software sends the signal to the framebuffer, there are internal hardware corrections which will further process the signal, specifically by gamma correcting it another 1.4 - That is, the signal is raised to the 1/1.4. Therefore, to get full correction, the software itself should first adjust the signal by raising it to the 1/1.8 power. (2.5/1.4 = 1.8) Thus the system gamma on a Macintosh is 1.8.

Note some graphics cards in Macintoshes may have their own software to change the standard gamma and Adobe Photoshop 3.0 is now released with a gamma control panel from Knoll software which allows the user to change the system gamma of their Macintosh. The 1.8 standard is still accepted as the universal Mac System Gamma, but users should be aware that a Mac can be set differently. The Knoll software control panel for the Mac rewrites the look up table, (LUT) with a value of g/2.5 where g is the gamma the user selects. Thus selecting 1.8 will rewrite the LUT with 1.8/2.5 = 1/1.4 - the default setting. (The values in the LUT are 1/1.4 and this is called a 1.4 correction)

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SGI

The SGI is similar to the Macintosh but instead of a hardware correction of 1.4 the SGI applies gamma correction of 1.7. Thus the system gamma for an SGI is 2.5/1.7 or roughly 1.5. Sometimes you may see that an SGI has a system gamma of 1.4. This calculation is made on the assumption that monitors have a response curve closer to a 2.4 power function.

SGI's also come with a utility to rewrite the internal hardware correction. These values are stored in a look up table. (LUT) and can be altered. The default is 1/1.7 as mentioned above. (The values in the table are 1/1.7, so we call this a 1.7 correction) Sometimes the value in the LUT may be referred to as the SGI system gamma. This is not the definition used on other platforms. Unlike the Mac gamma control panel, the SGI gamma utility will rewrite the LUT with the actual value set by the user. Gamma Definitions

Suns and PCs

Suns and PCs have no standard hardware correction (although certain graphics cards for these platforms may) and therefore their system gammas are roughly 2.5. (More about Sun's SX hardware and gamma)

Common graphics software such as Adobe Photoshop allows the user to set the gamma correction value they want. (In Photoshop it is found in Monitor Setup under Preferences under the File Menu.) More about software gamma correction

Exhibit C

REDACTED IN ITS ENTIRETY

Exhibit D

United States Patent [19]

Iida et al.

[11] **4,394,688**

[45] **Jul. 19, 1983**

[54] VIDEO SYSTEM HAVING AN ADJUSTABLE DIGITAL GAMMA CORRECTION FOR CONTRAST ENHANCEMENT

[75] Inventors: Hitoshi Iida, Bedford; Pay-Shin King, Newton, both of Mass.

[73] Assignee: Hamamatsu Systems, Inc., Waltham,

[21] Appl. No.: 296,068

[22] Filed: Aug. 25, 1981

358/37, 168, 169, 903, 111; 364/515

[56] References Cited

U.S. PATENT DOCUMENTS

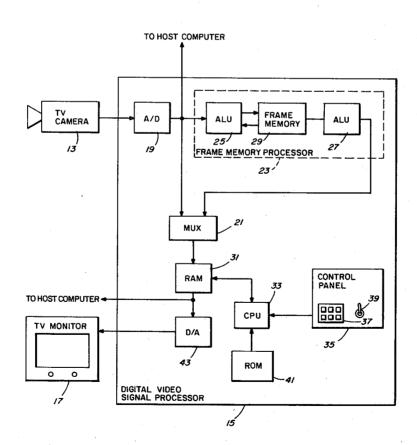
3,800,078	3/1974	Cochran et al 358/166 X
4,148,070	4/1979	Taylor 358/160
4,242,707	12/1980	Budai 358/160
4,335,427	6/1982	Hunt et al 358/166 X

Primary Examiner—Robert L. Richardson Attorney, Agent, or Firm—Irving M. Kriegsman

[57] ABSTRACT

A video system having a television camera, a digital video signal processor coupled to the output of the television camera and a television monitor coupled to the output of the digital video signal processor is disclosed. The digital video signal processor includes an analog to digital converter for converting analog video signals into video data, a frame memory processor for processing and temporarily storing the video data, a random access memory device in which the video data is altered in accordance with the contents of a tablelook-up temporarily written therein, a read only memory device containing a plurality of different table-lookups, each table-look-up containing data representing a different gamma correction, a central processing unit for obtaining a table-look-up from the read only memory device and then writing the table-look-up into the random access memory device and a manually operated control device coupled to the central processing unit for selecting which table-look-up is read out from the read only memory device and then written into the random access memory device and a digital to analog converter for converting the processed video data into analog video signals.

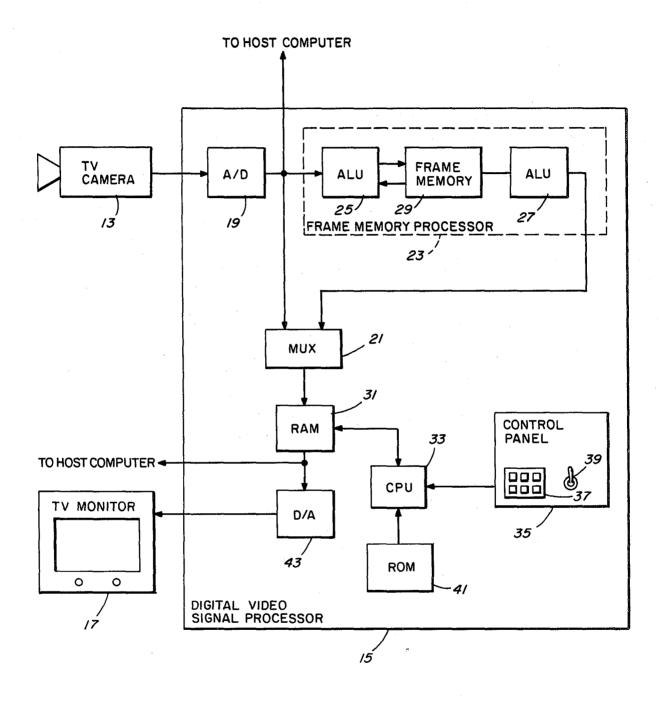
9 Claims, 1 Drawing Figure



U.S. Patent

Jul. 19, 1983

4,394,688



4,394,688

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VIDEO SYSTEM HAVING AN ADJUSTABLE DIGITAL GAMMA CORRECTION FOR CONTRAST ENHANCEMENT

BACKGROUND OF THE INVENTION

The present invention relates generally to video systems and more particularly to a video system which includes an adjustable digital gamma correction which is used to selectively emphasize or enhance the contrast 10 of a video picture over different regions of interest.

In a television system the relationship between the brightness of a portion of an object and the brightness of the corresponding portion of the image is generally expressed by the formula:

 $y=x^{\gamma}$

where y is the magnitude of the output signal, x is the magnitude of the input signal and y is the power, referred to as the gamma, to which x must be raised to be equal to y. In order to determine the overall gamma of a television system, the gamma of the television camera and the gamma of the television monitor or receiver are multiplied together. In an ideal or distortionless system 25 the overall gamma is 1.0. Unfortunately, in most all television cameras the gamma (which is determined by the composition of the photosensitive material in the vidicon) is between around 0.6 to 0.5 while in most all television monitors the gamma (which is determined by the composition of the phosphorescent material in the screen) is designed to be about 2.2. Consequently, the overall gamma is normally not equal to 1.0 and the contrast of the resulting image is somewhat distorted (i.e. in certain areas the contrast is greater than it should appear and in other areas the contrast is less than it should appear).

In order to correct for this distortion, a type of analog electrical circuit called a gamma correction circuit is normally incorporated into the system. The circuit provides a gamma which when multiplied together with the gamma of the television camera and the gamma of the television monitor produces an overall gamma in the system of around 1.0. These gamma correction circuits are normally built into the television camera or a control module for the television camera and are usually adjustable within a small range, such as from 0.85 to 1.0, to compensate for variations that may be present in the gamma of the particular vidicon tube used in the television camera. Once set to the particular value needed to produce an overall gamma of 1.0 (or as close to 1.0 as is possible), the gamma of the gamma correction circuit is generally not changed. However, in some closed circuit television systems used for surveillance purposes it is known to provide a knob or other manually adjustable means at the television monitor for ad- 55 justing the gamma produced by the gamma correction circuit for the purpose of intentionally distorting the contrast over areas of interest where the lighting is poor and the resulting image difficult to perceive. The amount of intentional distortion that can be produced, however, is limited to the small range of adjustability in the analog gamma correction circuit.

It is known to intentionally distort the contrast of an image formed in video microscope systems used in industrial and research applications to examine character- 65 istics and properties of very small objects in order to improve the visibility of the objects being examined by selectively manipulating the gain and offset knobs in the

video camera and the diaphragm and compensator settings in the microscope. An example of this technique of

contrast enhancement may be found in a publication Contrast Polarization entitled Video-Enhanced (AVEC-POL) Microscopy appearing in Cell Motility 1:275-289 (1981), Alan R. Liss, Inc.

It is also known to convert analog video signals into video data for temporary or permanent storage and/or digital signal image processing. One known type of digital signal image processing that is often performed to improve the quality of an image is noise reduction. An example of a known digital type image processing system may be found in U.S. Pat. No. 4,240,113, to Michael et al. Another example of a digital image processing system is described in Hamamatsu Systems Inc. Product Bulletin/2001 Rev. 2 2-81.

It is an object of this invention to provide a new and improved video system which is especially suited for use in, but not exclusively limited to, industrial and research applications.

It is another object of this invention to provide a novel method and system for selectively manipulating the contrast of a video image.

It is a further object of this invention to provide a video system having an adjustable digital gamma cor-

It is yet still another object of this invention to provide a novel method and system for selectively manipulating the contrast of a video image over low intensity

It is a further object of this invention to provide a novel method and system for selectively manipulating the contrast of a video image over high intensity areas.

It is yet still another object of this invention to provide a video system in which the gamma is adjustable and useable over a range from 0.1 or lower to 3.0 or higher.

It is another object of this invention to provide a system and method for digitally adjusting the gamma of a video system in order to provide contrast enhancement in selected areas of interest.

SUMMARY OF THE INVENTION

A video system constructed according to the teachings of the present invention includes a television camera, a digital video signal processor coupled to the output of the television camera and a television monitor coupled to the output of the digital video signal processor. The digital video signal processor includes an analog to digital converter for converting analog video signals from the television camera into video data, a random access memory device through which the video data is passed and in which the video data is altered in accordance with the contents of a table-lookup temporarily written therein, a read only memory device containing a plurality of different table-look-ups for use in the random access memory device, each tablelook-up containing data representing a different gamma correction, a central processing unit for obtaining a table-look-up from the read only memory device and then writing the table-look-up so obtained into the random access memory device in response to control signals applied thereto, a manually operated control device for applying control signals to the central processing unit for selecting which table-look-up is read out from the read only memory device and then written into the random access memory device and an analog to digital 4,394,688

converter for converting the processed video data into analog video signals. In the operation of the digital video system, the contrast of the video picture appearing the television monitor is adjusted for optimum viewing conditions over an area of interest by changing the 5 particular table-look-up in the random access memory device until the most favorable picture is produced.

The foregoing and other objects as well as many advantages of the invention will appear from the description to follow. In the description, reference is made 10 to the accompanying drawings which form a part thereof, and in which is shown by way of illustrating, specific embodiments for practicing the invention. These embodiments will be described in sufficient detail to enable those skilled in the art to practice the inven- 15 tion, and it is to be understood that other embodiments may be utilized and that structural changes may be made without departing from the scope of the invention. The following detailed description is, therefore, not to be taken in a limiting sense, and the scope of the 20 present invention is best defined by the appended

BRIEF DESCRIPTION OF THE DRAWINGS

In the drawings wherein like reference numerals rep- 25 resent like parts, the sole FIGURE is a simplified block diagram of a video system including a digital video signal processor constructed according to the principles of the present invention.

DETAILED DESCRIPTION OF PREFERRED EMBODIMENT

Referring now to the drawings, there is illustrated in the sole FIGURE a simplified block diagram of a video system constructed according to the teachings of the 35 present invention and identified generally by reference numeral 11.

Video system 11 includes a television camera 13 for generating analog video signals of an object or scene (or an optical image of an object or scene such as may be 40 formed by a microscope), a digital video signal processor 15 for digitally processing the video signals obtained from the TV camera 13 and a television monitor 17 for displaying an image of the processed video signals.

The digital video signal processor 15 includes an 45 analog to digital converter 19 for converting the analog video signals from TV camera 13 into video data. The video data from analog to digital converter 19 is transmitted to one of two inputs of a multiplexer 21 and to the input of a frame memory processor 23. The video 50 15, the particular table-look-up that is read out from the data from analog to digital converter 19 may also be transmitted to a host or main computer (not shown) for permanent storage and processing, if desired.

In the frame memory processor 23, video data of a single frame is temporarily stored and processed either 55 before or after storage. The frame memory processor 23 is shown in the FIGURE as including a pair or arithmetic logic units 25 and 27 and a frame memory 29. Frame memory processor 23 performs various known arithmetic type video signal processing functions, such as sum- 60 ming, averaging or differencing, the particular construction of frame memory processor 23 to perform these functions for purposes such as noise reduction not being considered a part of the invention and the particular components shown therein being for illustrative 65 purposes only. Frame memory 29 may comprise a pair of Intel chips number 2117. An example of a known frame memory processor capable of performing adding,

substracting and averaging type signal processing is described in the above referenced Hamamatsu Systems Inc. Product Bulletin/2001, Revision 2, 2/81. The output of the frame memory processor 23 is transmitted to the other input of multiplexer 21.

The video data from multiplexer 21, which is obtained either from analog to digital converter 19 or frame memory processor, is transmitted to a high speed random access memory device 31, where it is modified by a table-look-up which is temporarily written therein. Random access memory device 31 may comprise a pair of Fairchild chips number 93L422 and conventional associated logic circuitry. Random access memory device 31 is loaded with the table-look-up which is used to modify the video data from a central processing unit (CPU) 33 which controls the overall operations of the digital video signal processor 15 through signals sent over appropriate control lines (not shown). CPU 33 may be, for example, an Intel chip number 8080. The program for operating CPU 33 is stored in a memory (not shown) and instructions for executing the operating program are entered through a manually operated control panel 35 which includes a keyboard 37 and a joystick 39.

The table-look-up which is sent to random access memory device from CPU 33 is obtained from a bank of different table-look-ups permanently stored in a read only memory device 41 which is coupled to CPU 33. Each one of the table-look-ups in memory device 41 is 30 for a different gamma correction. For example, there may be a bank of thirty table-look-ups, with each tablelook-up having a different gamma correction, the smallest gamma correction table being 0.1 or smaller and the largest gamma correction table being 3.0 or larger. As can be appreciated, if the gamma resulting from TV camera 13 and TV monitor 17 is, for example, 1.32 a set of gamma correction tables having a range of 0.1 to 3.0 will enable the overall gamma to be changed from 0.132 to 3.966. Read only memory device 41 may comprise a pair of Intel chips number 2716 and associated logic circuitry.

The modified output data from random access memory device 31 is fed into a digital to analog converter 43 where it is converted into analog video signals. The modified output data may also be sent to the host computer, if desired. The image corresponding to the video output signals from digital to analog converter 43 is displayed on TV monitor 17.

In the operation of the digital video signal processor read only memory device 41 and written into the random access memory is controlled through the manually operated control panel 35. For example, the CPU 33 may be programmed so that the specific table-look-up written into the random access memory 31 from the read only memory 41 is determined by the angular position of the joy stick 39. Alternatively CPU 33 may be programmed so that different look-up-tables can be moved from read only memory 41 to random access memory 31 by depressing different keys or combinations of keys on the keyboard 37. As can be appreciated, the gamma correction applied to the video data can thus be very easily and very quickly changed over a range limited solely the number of different tables stored in the read only memory device 41 and the gamma values preselected for the individual tables.

In using the digital video signal processor 15 to manipulate contrast, data sent to multiplexer 21 either 4,394,688

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directly from analog to digital converter 19 or from frame memory processor 23 is fed from multiplexer 21 into random access memory 31, the particular data selected being controlled by CPU 33. The gamma correction table inserted into random access memory device 31 is than changed until the most favorable image is formed on the screen of the TV monitor 17. By changing the gamma correction over a portion of or over the entire range of table-look-ups, areas of interest in the resulting picture may be seen which might not otherwise be visually perceptible.

As is known, for certain applications such as viewing video pictures of X-rays, it is benefical to emphase contrast in low intensity areas while in other applications such as in video microscopy it is beneficial to emphasize contrast in high intensity areas. Thus, in certain instances low gamma corrections are desirable while in other instances high gamma corrections are desirable.

As is also known, at very high or very low gammas, 20 noise represents a problem that can very adversely effect the resulting picture. However, by processing the video data through frame memory processor 23, the noise can be easily reduced or substantially eliminated. Therefore, when using very high or very low gammas, 25 the video data should be processed for noise reduction in the frame memory processor 23.

The above description is considered as illustrative only of the principles of the invention. Further, since numerous modifications and changes will readily occur ³⁰ to those skilled in the art it is not desired to limit the invention to the exact construction and operation shown and described, and accordingly all suitable modifications and equivalents may be resorted to, falling within the scope of the invention as claimed.

What is claimed is:

- 1. A video system comprising a television camera for converting visual information into analog video signals, a digital video signal processor for digitally processing said anolog video signals and a television monitor for displaying an image of the processed analog video signals, said digital video signal processor including:
 - a. an analog to digital converter for converting the analog video signals into video data,
 - b. a first memory device for modifying the video data according to the particular contents contained therein,
 - c. a second memory device, said second memory device having stored therein a plurality of table- 50 look-ups, each table-look-up corresponding to a different gamma correction,
 - d. a central processing unit for controlling the operations of the digital video signal processor, said operations including reading out one of said table- 55 look-ups in said second memory device and writing said read data into said first memory device,

e. a manually operated control device for controlling the operations of the central processing unit, said control device including means for selecting the particular table-look-ups to be read by said central processing unit, and

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f. a digital to analog converter for converting the modified video data from said first memory device

into analog video signals.

2. The video system of claim 1 and wherein said first 10 memory device comprises a pair of high speed RAMS.

- 3. The video system of claim 1 and further including a frame memory processor for processing and/or storing a single frame.
- 4. The video system of claim 1 and wherein the means for selecting the particular table-look-up to be read by the central processing unit comprises a joystick.

5. A method of manipulating video picture data for the purpose of enhancing contrast over certain areas of

interest comprising:

- a. passing said video picture data through a first memory device which is arranged to modify the video picture data according to the contents of a tablelook-up contained therein,
- b. providing a second memory device having therein a plurality of different table-look-ups, each tablelook-up corresponding to a different gamma correction, and
- c. selectively reading out one of said table-look-ups from said second memory device and writing said table-look-up so read into said first memory device.
- 6. A digital video signal processor for processing analog video signals comprising:
 - a. an analog to digital converter for converting the analog video signals into video data,
 - b. a first memory device for modifying the video data according to the particular contents contained therein.
 - c. a second memory device having therein a plurality of table-look-ups, each table-look-up constituting a different gamma correction,
 - d. a central processing unit for moving one of the table-look-ups from the second memory device to the first memory device and
- e. control means for selecting which table-look-up is moved from the second memory device to the first memory device.
- 7. A digital video signal processor according to claim 6 and wherein said first memory device comprises a pair of high speed RAMS.
- 8. A digital video signal processor according to claim 6 and wherein including a frame memory processor for processing and/or storing a single frame.
- 9. A digital video signal processor according to claim 6 and wherein the means for selecting the particular table-look-up to be read by the central processing unit comprises a joystick.

Exhibit E

Case 1:06-cv-00738-SLR Document 111-6 Filed 01/18/2008 Page 2 of 5

THE OXFORD ENGLISH DICTIONARY

SECOND EDITION

Prepared by

J. A. SIMPSON and E. S. C. WEINER

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continuity (konti'njusti). [a. F. continuité (16th c.), ad. L. continuitat-em, f. continu-us: see -ITY.]

The state or quality of being continuous.

1. Of material things: The state or quality of being uninterrupted in extent or substance, of having no interstices or breaks; uninterrupted connexion of parts; connectedness, brokenness.

brokenness.

1543 [see 5]. 1570 DEE Math. Pref. Dj, Fyre and Ayre... will descend, when... their Continuitie should be dissoluted. 1607 Torsell. Four-f. Beasts (1673) 38 Inflaming the body, loosing the continuity of the parts. 1615 CROOKE Body of Man 307 Now there is no continuity betweene the vmbilicall veine and the hollow veine. 1646 Sir T. BROWNE Preud. Ep. II. i. 55 Continuity of parts is the cause of perspicuity. 1727-51 CHAMBERS Cycl., Continuity is usually defined, among schoolmen, the immediate cohesion of parts in the same quantum. 1804 WELLINGTON in Gurw. Dip. III. 59 The continuity of the frontier. 1813 BAKEWELL Introd. Geal. (1815) 52 Sometimes the continuity of rocks and strata is... broken. 1855 BAIN Senses & Int. 1. ii. \$17 (1864) 46 The continuity of the cord with the brain is necessary.

2. a. Of immaterial things, actions, processes, etc.: The state or quality of being uninterrupted in sequence or succession, or in essence or idea;

etc.: The state or quality of being uninterrupted in sequence or succession, or in essence or idea; connectedness, coherence, unbrokenness.

1603 HOLLAND Plutarch's Mor. 136 All that shall be, hath a stint and dependance of that which is, by a certeine continuitie, which proceedeth from the beginning to the end. 1751 HARRIS Hermer vii. (1786) 101 We may gain some idea of Time, by considering it under the notion of a transient continuity. 1830 W. IRVING SHEICH BR. I. 10 In travelling by land there is a continuity of scene, a connected succession of incidents that carry on the story of life. 1842 W. GROVE Corr. Phys. Forces Pref. (ed. 6) in The continuity of attention necessary for the proper evolution of a train of thought.

b. law or principle of continuity: the principle that all change, sequence, or series in nature is continuous, and that nothing passes from one

continuous, and that nothing passes from one state to another per saltum.

The phrase originated with Leibnitz. In 1687 he laid down as a general principle, that where there is continuity between data, such that one case continually approaches and at length loses itself in another, there will be a corresponding continuity in results or properties. For example, it is a property of the ellipse that all rays from the one focus are reflected from the curve to the other; in the parabola all such rays reflected at the curve are parallel; if there be given a series of ellipses continually approaching the parabola by the continuous increase of distance between the foci, the focal radii of these will continuously approach the relation of parallelism, so as at length to differ from it by less than any assignable amount. This was according to Leibnitz' a principle of general order', having its origin in the mathematical infinite, absolutely necessary in Geometry, but holding good also in Physics, because the Sovereign Wisdom, the source of all things, acts as a perfect Geometer, and according to a harmony that admits of no addition. In 1702 he referred to this principle as 'the law of continuity', and claimed that it operates in all natural phenomena; and in his Nowceaux Essais, he declared it to be part of his 'Law of Continuity' that everything in nature goes by degrees, and nothing per saltum.

[1687] Leibnitz Leitre à Mr. Bayle Wks. Erdm. 104. 1690

his Nouveaux Essais, he declared it to be part of his 'Law of Continuity' that everything in nature goes by degrees, and nothing per saltum.

[1687] LEIBNITZ LEITTE à Mr. Bayle Wks. Erdm. 104. 1690.

— Lettre à Mr. Arnauld ibid. 107 Chacune de ces substances contient dans a nature legem continuationis seriei suarum operationum. 1702. — Repl. aux Reft. de Bayle ibid. 80/2 Qu'il ne se rooz. Repl. aux Reft. de Bayle ibid. 80/2 Qu'il ne se recounte jamais rien, où la loi de la continuité (que J'ai introduite, et dont j'ai fait la première memor dans les Nouvelles de la République des Lettre de Mr. Bayle), et toutes les autres regles les plus exactes des Mathematiques soient violes. a 1716. — Nouv. Est. 11. xvi, Tout va par degrés dans la nature et rien par saut, et cette règle, à l'égard des changements, est une partie de ma loi de la continuité. J 1753 CHAMBERS Cycl. Suppl. s.v., An eminent mathematician has supposed what he calls a law of continuity to obtain in the universe, by which law every thing that is executed or done in nature, is done by infinitely small degrees. 1812-6 PLAYARIA Nat. Phil. (1819) 1. 271 When bodies, whether solid or fluid, act on on one another by impulse or percussion, in such a manner that their action is subject to the law of continuity. 1830 HERSCHEL Stud. Nat. Phil. 189 It prevents a breach of the law of continuity between transparent and opake bodies. 1841 J. R. Young Math. Dissert. ii. 74 That the angle changes at once from 90° to zero, is to admit so palpable a violation of the principle of continuity. 1837 TAIT & STEWART Muscan Muir. (1880) p. xii, We endeavour to show. that immortality is strictly in accordance with the principle of Continuity (rightly viewed).

C. equaction of continuity, in Hydrodynamics: the equation conpecting the rate of change of

c. equation of continuity, in Hydrodynamics: the equation connecting the rate of change of density of a fluid within any closed surface constantly full of fluid with the flow of fluid

through the surface.

1836 T. Webster Equilibr. & Motion of Fluids. 1880
HAUCHTON Phys. Geog. iii. 141. 1882 MINCHIN Unipl.
Kinemat. & Minchin Unipl.

HAUGHTON Phys. Geog. In. 141.

Kinemat. §93.

3. The state or quality of being continuous in rare.

"" Wee need not

3. I ne state or quality of being continuous in time; uninterrupted duration. rare.

1646 Sir T. Browne Pseud. Ep. 1v. xiii, Wee need not have recourse unto any starre but the Sunne and the continuity of its action. 1840 Mss. Browning Drama of Exile Poems 1850 I. 27 Their stedfast continuity of gaze. 1841 Brewster Mart. Sc. 11. iv. (1856) 146 A painful disease, which had its origin in the severity and continuity of his studies.

his studies.
4. a. quasi-concr. A continuous or connected

whole; a continuous or connected whole; a continuous or unbroken course or series. (Of material or immaterial things.)

1601 HOLLAND Pliny II. 423 Running throughout one continuity without interruption. a 1619 FOTHERBY Atheom.

II. ix. §3 (1622) 296 All magnitudes and continuities are

deduced from one originall prick. 1644 MILTON Areop. (Arb.) 70 When every stone is laid artfully together, it cannot be united into a continuity, it can but be contiguous in this world. 1809-10 COLERIDGE Friend (1865) 219 A chain that ascends in a continuity of links.

b. A part continuous with something else.

1809 W. IRVING Knickerb. (1861) 248 The New-Netherlands. a continuity of the territory taken possession of by the Pilgrims, when they landed on Plymouth Rock. 5. solution of continuity: the fact or condition of being or becoming discontinuous; fracture, rupture, breakage, break. Orig. used of wounds, etc. in an animal body; thence also in

other senses.

1543 Traherron tr. Vigo's Chirurg. (1586) 12 The heart can not suffer solution of continuitie without death. 1661 Bramhall. Just Vind. ii. 14 Schisme is an exteriour breach, or a solution of continuity in the body Ecclesistick. 1707 Curios. in Husb. & Gard. 77 The Solution of Continuity may hinder the Juice from mounting. 1790 Buske Fr. Rev. 24 With what address this temporary solution of [historical] continuity is kept from the eye. 1877 Tyndall in Daily News 2 Oct. 2/5 We are brought without solution of continuity into the presence of problems, which lie entirely outside the domain of physics.

6. a. Cinemat. A detailed scenario for a cinema film: also, the maintenance of consistency of

film; also, the maintenance of consistency or a continuous flow of action in successive shots or scenes of a cinema or television film. orig. U.S.

scenes of a cinema or television film. orig. U.S. 1921 Collier's 25 June 26/3 It is a hobby of mine never to pay no attention to a continuity, as it only gets a man balled up. 1926 G. F. BUCKLE Mind & Film 1. 18. 1 am deliberately using the word scenario here instead of continuity (which is the correct name for the working script) in order to avoid confusion when discussing faults in the scenario which result in bad continuity in the film. 1927, etc. [see sense 7 below]. 1930 Timer 12 July 4/6 Mr. Knoblock became joint author...of the scenario of the talking film and he also wrote the 'continuity' or stage directions. 1946 F. Scorr FITZGERALD Let. 11 May (1964) 116 I've written a really brilliant continuity. It seems to be a last life line that Hollywood has thrown me. 1940 War Illustr. 12 Jan. p. ii. 2 Touching this matter of 'continuity' but see a word from film-technique. 1951 I, WYDHAM' Day of Triffid vi. 116 Sandra is our professional remembrancer—continuity is her usual work.

b. A series of linking announcements,

A series of linking announcements, interpolations, or the like in a radio programme

interpolations, or the like in a radio programme or broadcast; the maintenance of a continuous sequence in broadcasting (see quot. 1941).

1934 B.B.C. Year-Bk. 107 The Ceremony of the Keys is a good example of a broadcast which requires elaborate technical arrangements to ensure 'continuity'. 1941 B.B.C. Gloss. Broadc. Terms 7 Continuity, (1) structure of verse or prose, linking sections of a dramatic or of a magazine programme; (2) announcements or interlude material filling interviews between programmes in a sequence of programmes. 1968 Littener 4 Jan. 28/2 All the Best. is. full of limp interviews and badly written continuity.

7. attrib., as continuity announcement, announcer, clerk; continuity girl Cinemat., a worman who is responsible for ensuring that

woman who is responsible for ensuring that there are no discrepancies of detail between linked scenes filmed at different times; continuity studio (see quot. 1941); continuity sulte (see quot. 1962); continuity title, an explanatory title inserted into a film to ensure continuity; continuity writer, a writer of

explanatory title inserted into a film to ensure continuity; continuity writer, a writer of continuity titles.

1962 A. NISBETT Technique of Sound Studio i. 17 The output of individual studios is ...linked together by ... "continuity announcements. Ibid., "Continuity girl. 1933. "I. Hav." & A. Armstranon. Order are Orders. II. 46 Meet Miss Marigold, my secretary and "continuity girl. 1934 R. Manwell. Film 1. vi. 68 The director is the admitted coordinator of the actors' work, with the continuity girl killing the details. 1970 Daily Tel. (Colour Suppl.) 3 July 23 The world of folding chairs, megaphones and continuity girls is put firmly behind him when he turns to domestic filmmaking. 1941 B.B.C. Gloss. Broadc. Terms 7 "Continuity studio, small studio from which an announcer, supervising the running of a sequence of programmes, makes opening and closing announcements and interpolates interlude material when required. 1957 B.B.C. Handbh. 52 The machines are started and stopped by remote control. from a "continuity suite in the case of reproductions. 1962 A. NISBETT Technique of Sound Studio 246 Continuity suite, a centre through which programmes are routed or where they are reproduced to build a particular service ready for feeding to a transmitter. 1953 K. Reisz Film Editing 279 "Continuity title, title designed to bridge a break in the pictornal continuity. 1921 Moving Pict. Stories 9 Dec. 28/2 He knows that the "continuity writer is going to make certain changes. 1928 Sunday Express 18 Mar. 4/3 The necessity for such titles as "Time went on, and brought changes in the circus' would bring most continuity writers into diagrace.

continuo (kən'tin(j)u:20). Mus. = BASSO

continuo (kənˈtɪn(j)u:əu). Mus. continuo; also, an instrument or instruments

continuo; also, an instrument or instruments playing this.

1724 Short Explic. For. Words Mus. Bks. 24 Continuo, for this see the Words Daiso continuo. 1740 Grassineau tr. S. de Brossard's Mus. Dict. 44 Continuo, signifies the thorough Bass., as Baso continuo is the continual or thorough Bass. 1879 Grove Dict. Mus. 1. 151/2 In the score of the Matthäus Passion' of Bach... in the recitative a single line and figures is given for the 'continuo' alone. 1942 E. Blom Music in Eng. iii. 41 These harmonic instruments thus began casually to take on the function they or their descendants assumed quite regularly in the next two centuries under the name of continuo.

CONTINUOUS

continuous (kən'tınju:əs), a. [f. L. continu-us hanging together, uninterrupted (f. contin-ēre in intr. sense 'to hang together,' etc.) + -ous.]

1. a. Characterized by continuity; extending in space without interruption of substance; having no interstices or breaks; having its parts in

no interstices or breaks; having its parts in immediate connexion; connected, unbroken.

1673 Grew Anat. Plants II. iii. §3 It is Compounded of two Bodies. The one Parenchymous; Continuous throughout; yet somewhat Pliable without a solution of its Continuity. 1704 Newron Optics II. II. (1782) IV. 148 The dark intervals must be diminished, until the neighbouring rings become continuous, and are blended. 1795 SOUTHEY Joan of Are VII. 6 Round the city stretch'd Their line continuous, massy as the wall Erst by the fearful Roman. raised. 1859 Darwin Orig. Spec. Xii. (1873) 320 In most cases the area inhabited by a species is continuous. 1879 LOCKYRE Elem. Atton. VI. 28 If we light a match and observe its spectrum, we find that it is continuous—that is, from red through the whole gamut of colour to the visible limit of the violet. 1881 Maxwell. Electr. & Magn. 1. 6 Without describing a continuous line in space.

b. In unbroken connexion with joined

b. In unbroken connexion with; joined continuously to; forming one mass with.

continuously to; forming one mass with.

1692 RAY Distal. World XI. V. (1732) 207 Anciently
continuous with Malacca. 1700 S. PARKER Six Philos. Ess. 95
The Superficies whereto it was continuous, etc. 1879
HARLAN Eyesight ii. 25 The mucous membrane of the eye is
continuous with the skin.

† c. fig. Obs.

1642 FULLER Holy & Prof. St. IV. iii. 252 They were so
contiguous and near in kinred, they might not be made
continuous (one flesh) in marriage.

2. a. Of immaterial things, actions, etc.:
Uninterrupted in time. sequence, or essence:

Uninterrupted in time, sequence, or essence; going on without interruption; connected, unbroken

unbroken.

1731 Harris Hermes II. (1841) 187 Continuatives...

1731 Consolidate sentences into one continuous whole. 1832 Nat. Philos., Electro-Magnet. xi. §176. 60 (Useful Knowl. Soc.) The currents transmitted by perfect conductors are continuous; that is, their intensity is either constant, or varies insensibly during two consecutive instants. 1867 SMYTH Sailor's Word-bk., Continuous service men, those seamen who, having entered for a period, on being paid off, are permitted to have leave, and return to the flag-ship at the port for general service. 1867 FEERMAN Norm. Conq. 1. App. (1876) 700 A continuous siege of six months. 1875 JOWETT Plato (ed. 2) V. 131 The power of abstract study or continuous thought is very rare. 1878 TAIT & STEWART Unseen Univ. VII. §212 Which will explain the continuous life of the universe as well as its continuous energy.

b. Gram. Of verb tense-forms or aspect: denoting continuous action (see quots.). Cf.

denoting continuous action (see quots.).

b. Gram. Of verb tense-forms or aspect: denoting continuous action (see quots.). Cf. EXPANDED ppl. a. 2 b and PROGRESSIVE a. 3 h. 1887 N.E.D. s.v. be v., B 15 With the present participle, forming continuous varieties of the tenses. 1892 H. Sweet New Eng. Gram. I. to 2 Long tenses may be either continuous or recurrent, denoting repetition, habit, etc. Thus we have a continuous present in he lives in the country, a recurrent present in he goes to Germany twice a year. 1898 J. C. NESPIELD Eng. Gram. v. 58 Continuous Itense]. denotes that the event (in Present, Past, or Future time) is still continuing, and is not yet completed; as, 1 am loving, 1 was loving, 1 shall be loving, 1 shall be loving, 1 shall be loving, 1 shall be down for implies habit, eg. 1 thall be dning alone all week. 1947 W. S. Allen Living Eng. Struct. 70 It might. be shown that as a continuous tense describes an action as it is taking place, the Present Continuous is the only real present tense we have in English. 1965 F. R. PALMER Linguistic Study Eng. Verb iv. 59 Every second pair in the paradigm. is progressive. There is no obvious name for the category... The terms 'continuous' and 'noncontinuous' are sometimes used. 1965 R. Quink et al. Comprehensive Gram. Eng. Long. 197 The progressive aspect (sometimes called the durative or continuous aspect) indicates a happening in progress.

3. technically. continuous aspects the progress of the continuous of the continuous of the durative or continuous aspect on dicates a happening in progress.

3. technically. continuous assessment (Educ.), the evaluation of a pupil's progress throughout the course of study, based on course-work as well as, or instead of, examinations; see also ASSESSMENT 5b; continuous brake, a continuous series of carriage brakes controlled from one point, action, unconsider corriers or wheat in a training continuous action. acting upon every carriage or wheel in a train; continuous consonants, those which are capable of prolonged enunciation (opposed to explosive); continuous creation, creation viewed being a continuous process and not a single as being a continuous process and not a single act at a particular time; spec. the view that the universe is in a steady state, new systems being formed continually to replace those that disappear; continuous-flow, used attrib. designating a system, device, etc., in which a fluid or other material flows continuously; continuous function (Math.), a function that varies continuously and whose differential varies continuously, and whose differential coefficient therefore never becomes infinite; continuous impost: see IMPOST; continuous kiln (see quot. 1010); continuous miner (see quot. (see quot. 1910); continuous miner (see quot. 1967); continuous process, an industrial process which operates without interruption (opp. batch process); continuous spectrum, 'a spectrum not broken by bands or lines, but having the colors shaded into each other continuously, as that from an incandescent solid or liquid, or a gas under high pressure' (WEBSTER 1890) (cf. quot.



1879 for sense 1 a above); continuous stationery (see quot. 1942); continuous stem (Bot.), one without articulations; continuous style, in Gothic Architecture, a style in which the mullions of a window are continued in the tracery, as distinguished from the geometrical style of earlier Gothic; continuous tone (see quot. 1968); continuous variation, in Biol. (see quot. 1961); continuous voyage, a voyage which, though interrupted by stops at ports or otherwise, is regarded as a single voyage in reference to the purpose for which it was undertaken (e.g. the consignment of goods materials); continuous wave.

otherwise, is regarded as a single voyage in reference to the purpose for which it was undertaken (e.g. the consignment of goods or materials); continuous wave, an electromagnetic (esp. radio) wave having constant amplitude and intensity; also attrib.

1959 15 to 18: Rep. Cent. Advisory Council for Educ. (Eng.) (Ministry of Educ.) 1 xxv. 281 Some Institutes of perfection; the live eyes were a continuous creation. 1941 C. Sinose Short Hist. Sci. v. 140 Averroes believed, not in a single act of creation, but in a "continuous creation. 1945 P. Hovite Nature of Universe. v. 107 With continuous creation the apparent contradiction between the expansion of the Universe and the requirement that the background material shall. condense into galaxies is completely or adio-stars fooding the general shall condense into galaxies is completely or adio-stars fooding the general shall condense into galaxies is completely or adio-stars fooding the general shall condense into galaxies is completely or adio-stars fooding the general shall condense into galaxies is completely or adio-stars fooding the general shall condense into galaxies is completely or adio-stars fooding the general shall condense into galaxies is completely or adio-stars fooding the general shall condense into galaxies is completely or adio-stars fooding the general shall condense into galaxies is completely or adio-stars fooding the general shall condense into galaxies in the general shall general

continuously (kən'tınju:əslı), adv. [f. prec. + continuous manner:

uninterruptedly, without break; continually, constantly.

1678 CUDWORTH Intell. Syst. 167 (R.) Which incorporates the newly received nourishment, and joins incorporates the newly received nourishment, and joins it continuously with the preexistent parts of flesh and bone. 1836 FOSTER in Life & Corr. (1846) II. 94 He spoke continuously for a considerable time. 1875 LYELL Princ. Gool. I. II. xxv. 623 These may sometimes mantle continuously round the whole mass. 1879 Nature 20 Nov. 88 A body which is changing its speed every. hundredth part of a moment or what we call continuously. 1887 Maxwell Electr. & Magn. I. 6 A quantity is said to vary continuously, if, when it passes from one value to another, it assumes all the intermediate values.

continuousness (kən'tınju:əsnıs). [f. as prec. + -NESS.] The state or quality of being continuous;

continuity.

1803 W. TAYLOR in Monthly Mag. XVI 224 These two narratives are drawn up with that continuousness, that artiess wondering honesty, which might be expected. 181-19 DARWIN in Adm. Man. Space-time. continuum, see SPACE-TIME. Also attrib., as continuum mechanics, theory; continuum hypothesis Math., the hypothesis that there is no transfinite cardinal between the cardinal of the set of positive integers and that of the set of real numbers.

the set of positive integers and that of the set of real numbers.

1650 Sir T. Browne Pseud. Ep. 11. i. (ed. 2) 40 The fusible salt draws the earth and infusible part into one continuum. 1677 Hale Prim. Orig. Man. 1v. 1v. 327 The admirable accommodation of the several Parts of the Humane Body to make up one Continuum: 1865 Grore Plato 1. i. 1 There could be no continuum: each numerical unit was distinct and separated from the rest by a portion of vacant space. or 1878 Lewes Study Psychol. (1879) 133 To these animals (the wolf and dog) the external world seems a continuum of scents, as to man it is a continuum of sights. 1886 J. Ward in Encycl. Brit. XX. 5113 (Psychology) All possible sensations of colour, of tone, and of temperature constitute as many groups of qualitative continua. 1903 B. RUSSELL. Princ. Math. 1. xxxvi. 297 Cantor proves that any series M satisfying the above conditions is ordinally similar to the number-continuum, i.e. the real numbers from o to 1, both inclusive. 1905 Westim. Gaz. 14 Aug. 2/1 We must know more of the properties of the ponderable atoms moving in the continuum which fills all space. 1924 W. B. SELBIE Psychol. of Relig. iv. 91 There are no doubt good physiological reasons for these phenomena of the sensory continuum. 1938 Proc. Nat. Acad. Sci. XXIV. 101 The continuum hypothesis of Georg Cantor. 1951 C. Tax ESDEL. in Jrnl. Math. Pures et Appliquées XXX. 116 The notion of an isotropic function. has been much employed in continuum mechanics. Ibid. 150 Continuum theory, being independent of any sort of deterministic mechanics of the ultimate particle, serves as a general guide with which any molecular model must be consistent. 1953 A. E. SMAILES independent of any sort of deterministic mechanics of the ultimate particle, serves as a general guide with which any molecular model must be consistent. 1953 A. E. SMAILES Geogr. Towns 33 There is no longer either socially or physically a simple clear-cut dichotomy of town and country, rather it is an urban-rural continuum. 1953 A. A. FRAENKEL Abstract Set Theory ii. 152 The assumption that, for every transfinite 1, 2' is the cardinal next larger than 1, is called the generalized continuum hypothesis. 1955 W. NOLL in frml. Rational Meth. & Anal. IV. 12 (heading) Basic concepts of continuum mechanics. 1962 L. J. Conten Diversity of Meaning (1966) i. 12 Descriptive studies are also needed to abstract and record the pivotal stages in a continuum of change. 1964 Proc. Nat. Acad. Sci. 1.1. 109 Since our additional axiom is quite readily acceptable to most mathematicians... one can regard the unprovability of the Continuum Hypothesis as firmly established.

contir-, obs. f. COUNTER-, e.g. in contirmont.

'cont-line. [Of uncertain derivation: it has been suggested that cont is a variant of CANT sb.1]

The spiral intervals formed between the strands of a rope, by their being twisted together'

1848 G. BIDDLECOMBE Art of Rigging 10. 1874 KNIGHT Dict. Mech. 612 Cont-line. in worming.. is filled up with spun yarn or small rope, which brings the rope so treated to a nearly cylindrical shape.

a nearly cylindrical shape.

2. "The space between the bilges of two casks stowed side by side'. 1867 SMYTH Sailor's Word-bk

conto ('konteu). In 6-7 counto. [Pg. conto = It. conto, OF. cunte, F. compte:—L. computus: see COUNT sb.] In Portuguese, a million; hence, short for a million reis, worth in Portuguese currency about £220, in Brazilian a little more than half that amount.

than half that amount.

1601 Hakluyt It. Galvano's Discov. World (1862) 14 He neuer. Left off to raise and to augment the yerely rent vnto a counto. 1836 SIMMONDS Dist. Trade Prod., A Portuguese word for million; a conto of reis (1000 millres) is usually expressed thus 1000\$5000. 1889 Times (weekly ed.) 13 Dec. 15/1 The sum of 5,000 contos (6/55,000). 1890 Daily News 25 Jan. 5/5 The money being subscribed in Brazil. The capital is stated to be 200,000 contos of reis, or over 20 millions sterling. 1891 Scot. Leader 13 May 6 (Lisbon) 1200 contos of reis of new silver money coined in virtue of last Friday's decree.

contoid ('kontoid), a. and sb. Linguistics. [f. cont., shortening of Consonant sb. + -oID.] A. adj. Consonant-like; of consonantal character; esp. as contrasted with vocoid a. B. sb. A speech

esp. as contrasted with vocoid a. B. sb. A speech sound of the consonantal type.

1943 K. L. Pike Phonetics v. 78 Vocoid and contoid groups are strictly delineated by the articulatory and acoustic nature of sounds. Ibid. vii. 143 Contoids include stops, fricative nasals, lateral resonant orals, and central fricative orals. 1957 H. J. ULDALL in Hjelmslev & Uldall Outl. Glossematics 49 It is possible to regard long vocoids and contoids as manifestations of chains of identical vowels or consonants. 1958 C. F. Hockett Course Mod. Ling. vii. 67 A contoid is a sound involved with the vocal tract, or else a complete interruption of the air stream. Use of the terms 'vocoid' and 'consonant' as labels for structurally defined classes of phonemes in specific languages. Ibid. defined classes of phonemes in specific languages. *Ibid.*, Consonant-like or contoid effects.

CONTORING TO THE CONTROL OF THE CONT

numismatists to certain brass pieces of Nero and other Roman emperors, the purpose of which is

Uncertain.

1833-5 T. D. Fosbroke Encycl. Antiq. (1843) 973
Contorniates... are mostly between two and three inches [in]
diameter. 1850 LEITCH fr. Muller's Anc. Art § 207. 108 The
contorniati distributed at public games. 1839 S. W.
STEVENSON Dict. Rom. Coins s.v., All writers appear...to
agree in considering that contorniates were not of the nature
and value of money... All contorniates are of brass.

con'torniated, con'tourniated, ppl. a. = prec. adi.

1727-51 CHAMBERS Cycl. s.v., All we have remaining of these contourniated medals, seem to have been struck about the same time. 1730-6 Bailey (folio), Contourniated. 1823

|| contorno (kon'torno). [It. contorno circuit, CONTOUR, f. contornare to turn together, compass about, put a thing round another (cf. med. L. contornare to round off well), f. L. con-+ tornare to turn in a lathe, round off, make round, f. tornus a turning-lathe.] Contour,

outline of a statue or other work of art.

1758 Johnson Idler No. 76 P3 His mouth full of .. the sublimity and grand contorno of Michael Angelo. 1781 MAD. D'ABRIAY DIAPY I. 325 For a background and contorno, who comes up to Mrs. Thrale?

contorsion, obs. form of contortion.

contorsive (kən'tə:siv), a. [f. L. contorsus, alleged variant of contortus (see CONTORT) +

alleged Variant of contoring specification, 1-1VE.] Of contorting quality or tendency, 1819 H. Busk Vestriad 1. 500 His eye contorsive bent thousand ways. 1819 — Dessert 711 Or with scored viscer contorsive rue The deleterious trash that vintners brew.

†con'tort, ppl. a. Obs. [ad. L. contort-us pa. pple: see next.] Twisted, contorted. 1570 LEVINS Manip. 173 Contort, contortus

contort (kən'to:t), v. [f. L. contort- ppl. stem of

contorquere, f. con- + torquere to twist.]

1. trans. To twist, twist together or round itself; to draw awry; to distort greatly by

itself; to draw awry; to distort greatly by twisting.

1622 [see CONTORTED]. c1715 CHEYNE (J.), Spires contorted into small spheres. 1736 AMORY Buncle (1770) I. 193 These.. fleshy fibres are contorted and bound about with...spiral ramifications. of the nerves. 1846 HAWTHORNE Mosses I. I. 10 The variety of grotesque shapes into which apple-trees contort themselves. 182-9 Toop Cycl. Anal. IV. 948/1 The cord is thereby contorted into a spiral. 1855 Bain Sense & Int. II. ii. \$2 (1864) 121 The features are violently contorted. 1879 Lockyer Elem. Astron. iii. 79 The sedimentary rocks have been...bent, contorted, or twisted to an enormous extent.

fig. 1836-7 Sir W. Hamilton Metaph. (1877) I. xi. 197 Contorted from their established signification. 1864 Bowen Logic vii. 192 Both halves of the reasoning are contorted. †2. To hurl forth as a missile or argument. Obs.

c 1562 ABP. PARKER Def. Priests Marriages 165 For it may be well verified of you that ye contort to another: He that is once ouer his shoes, forceth not afterward how deepe he wards in the move. wade in the myer.

contorted (kən'to:tid), ppl. a. [f. prec. + -ED.]

1. Twisted, esp. twisted together or round itself; drawn awry or out of shape by a twisting action.

Exhibit F

EIGHTH

INTERMEDIATE ALGEBRA

EDITION

Margaret L. Lial
American River College

John Hornsby University of New Orleans



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OBJECTIVE find the arithmetic mean (average) of a group of numbers. The arithmetic mean, or average, of a group of numbers is defined as the sum of all the numbers divided by the number of numbers.

Arithmetic Mean or Average

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10

The arithmetic mean, or average, of a group of numbers is symbolized \bar{x} and is found by dividing the sum of the numbers by the number of numbers. That is,

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}.$$

Here the values of x_i represent the individual numbers in the group, and n represents the number of numbers.

EXAMPLE 6 Finding the Arithmetic Mean or Average

The following table shows the number of companies listed on the New York Stock Exchange for each year during the period 1990–1996. What was the average number of listings for this seven-year period?

Year	Number of Listings	
1990	1774	
1991	1885	
1992	2088	
1993	2361	
1994	2570	
1995	2675	
1996	290 <i>7</i>	

Let $x_1 = 1774$, $x_2 = 1885$, and so on. Since there are 7 numbers in the group, n = 7. Therefore,

Source: New York Stock

Exchange.

$$\bar{x} = \frac{\sum_{i=1}^{7} x_i}{7}$$
=\frac{1774 + 1885 + 2088 + 2361 + 2570 + 2675 + 2907}{7}
= 2323 \tag{(rounded to the nearest unit)}.

The average number of listings for this seven-year period was 2323.